OCR Maths M2

Topic Questions from Papers

Projectiles

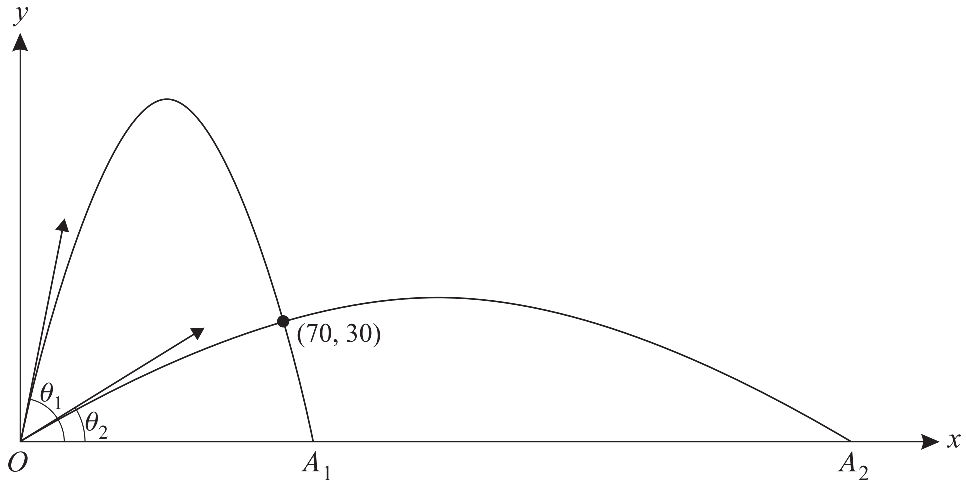
1 A particle is projected horizontally with a speed of $6\,\text{m}\,\text{s}^{-1}$ from a point 10 m above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground. [6]

(Q2, June 2005)

2 A particle is projected with speed $49\,\text{m}\,\text{s}^{-1}$ at an angle of elevation θ from a point O on a horizontal plane, and moves freely under gravity. The horizontal and upward vertical displacements of the particle from O at time t seconds after projection are x m and y m respectively.

(i) Express x and y in terms of θ and t, and hence show that

$$y = x\tan\theta - \frac{x^2(1+\tan^2\theta)}{490}.$$ [4]



The particle passes through the point where $x = 70$ and $y = 30$. The two possible values of θ are θ_1 and θ_2, and the corresponding points where the particle returns to the plane are A_1 and A_2 respectively (see diagram).

(ii) Find θ_1 and θ_2. [4]

(iii) Calculate the distance between A_1 and A_2. [5]

(Q8, June 2005)

3 A golfer hits a ball from a point O on horizontal ground with a velocity of $50\,\text{m}\,\text{s}^{-1}$ at an angle of $25°$ above the horizontal. The ball first hits the ground at a point A. Assuming that there is no air resistance, calculate

(i) the time taken for the ball to travel from O to A, [3]

(ii) the distance OA. [2]

(Q2, Jan 2006)

4 A stone is projected horizontally with speed $7\,\text{m}\,\text{s}^{-1}$ from a point O on the edge of a vertical cliff. The horizontal and upward vertical displacements of the stone from O at any subsequent time, t seconds, are x m and y m respectively. Assume that there is no air resistance.

(i) Express x and y in terms of t, and hence show that $y = -\frac{1}{10}x^2$. [4]

The stone hits the sea at a point which is 20 m below the level of O.

(ii) Find the distance between the foot of the cliff and the point where the stone hits the sea. [2]

(iii) Find the speed and direction of motion of the stone immediately before it hits the sea. [6]

(Q6, Jan 2006)

5 A small ball is projected at an angle of 50° above the horizontal, from a point A, which is 2 m above ground level. The highest point of the path of the ball is 15 m above the ground, which is horizontal. Air resistance may be ignored.

(i) Find the speed with which the ball is projected from A. [3]

The ball hits a net at a point B when it has travelled a horizontal distance of 45 m.

(ii) Find the height of B above the ground. [6]

(iii) Find the speed of the ball immediately before it hits the net. [4]

(Q7, June 2006)

6 A missile is projected with initial speed $42\,\text{m}\,\text{s}^{-1}$ at an angle of 30° above the horizontal. Ignoring air resistance, calculate

(i) the maximum height of the missile above the level of the point of projection, [3]

(ii) the distance of the missile from the point of projection at the instant when it is moving **downwards** at an angle of 10° to the horizontal. [11]

(Q8, Jan 2007)

7 Calculate the range on a horizontal plane of a small stone projected from a point on the plane with speed $12\,\text{m}\,\text{s}^{-1}$ at an angle of elevation of 27°. [4]

(Q2, June 2007)

8 A ball is projected from a point O on the edge of a vertical cliff. The horizontal and vertically upward components of the initial velocity are $7\,\text{m}\,\text{s}^{-1}$ and $21\,\text{m}\,\text{s}^{-1}$ respectively. At time t seconds after projection the ball is at the point (x, y) referred to horizontal and vertically upward axes through O. Air resistance may be neglected.

(i) Express x and y in terms of t, and hence show that $y = 3x - \frac{1}{10}x^2$. [5]

The ball hits the sea at a point which is 25 m below the level of O.

(ii) Find the horizontal distance between the cliff and the point where the ball hits the sea. [3]

(Q4, June 2007)

9 A ball is projected with speed $12\,\text{m}\,\text{s}^{-1}$ at an angle of elevation of $55°$ above the horizontal. At the instant when the ball reaches its greatest height, it hits a vertical wall, which is perpendicular to the ball's path. The coefficient of restitution between the ball and the wall is 0.65. Calculate the speed of the ball

(i) immediately before its impact with the wall, [2]

(Q1, Jan 2008)

10 A missile is projected from a point O on horizontal ground with speed $175\,\text{m}\,\text{s}^{-1}$ at an angle of elevation θ. The horizontal lower surface of a cloud is 650 m above the ground.

(i) Find the value of θ for which the missile just reaches the cloud. [3]

It is given that $\theta = 55°$.

(ii) Find the length of time for which the missile is above the lower surface of the cloud. [5]

(iii) Find the speed of the missile at the instant it enters the cloud. [4]

(Q7, Jan 2008)

11 A golfer hits a ball from a point O on horizontal ground with a velocity of $35\,\text{m}\,\text{s}^{-1}$ at an angle of θ above the horizontal. The horizontal range of the ball is R metres and the time of flight is t seconds.

(i) Express t in terms of θ, and hence show that $R = 125\sin 2\theta$. [5]

The golfer hits the ball so that it lands 110 m from O.

(ii) Calculate the two possible values of t. [5]

(Q4, June 2008)

12

Two small spheres A and B of masses 2 kg and 3 kg respectively lie at rest on a smooth horizontal platform which is fixed at a height of 4 m above horizontal ground (see diagram). Sphere A is given an impulse of 6 N s towards B, and A then strikes B directly. The coefficient of restitution between A and B is $\frac{2}{3}$.
the speed of B after it has been hit by A is $2\,\text{m}\,\text{s}^{-1}$.

Sphere B leaves the platform and follows the path of a projectile.

(ii) Calculate the speed and direction of motion of B at the instant when it hits the ground. [7]

(Q7, June 2008)

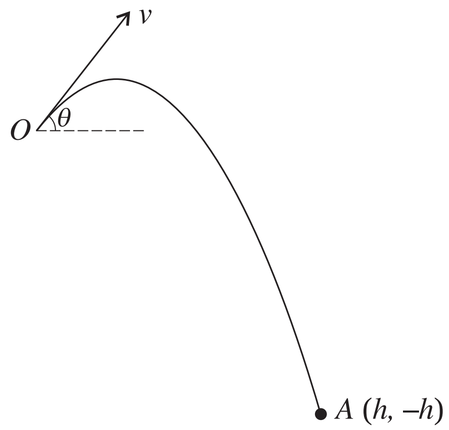
13 A stone is projected from a point on level ground with speed $20\,\text{m}\,\text{s}^{-1}$ at an angle of elevation of $\theta°$ above the horizontal. When the stone is at its greatest height it just passes over the top of a tree that is 17 m high. Calculate θ. **[4]**

(Q1, Jan 2009)

14 A particle is projected from a point O with speed $v\,\text{m}\,\text{s}^{-1}$ at an angle of elevation θ above the horizontal and it moves freely under gravity. The horizontal and upward vertical displacements of the particle from O at any subsequent time, t seconds, are x m and y m respectively.

(i) Express x and y in terms of θ and t, and hence show that

$$y = x\tan\theta - \frac{4.9x^2}{v^2\cos^2\theta}.$$ **[4]**



The particle subsequently passes through the point A with coordinates $(h, -h)$ as shown in the diagram. It is given that $v = 14$ and $\theta = 30°$.

(ii) Calculate h. **[4]**

(iii) Calculate the direction of motion of the particle at A. **[5]**

(iv) Calculate the speed of the particle at A. **[2]**

(Q6, Jan 2009)

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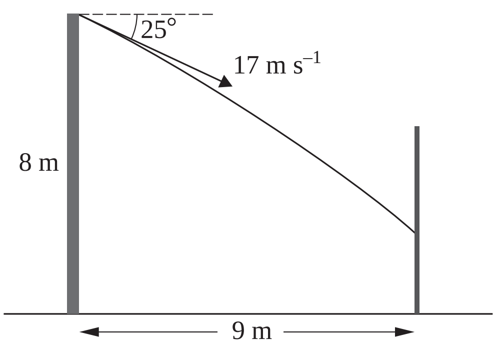

A ball is projected with an initial speed of $17\,\text{m}\,\text{s}^{-1}$ at an angle of $25°$ below the horizontal from a point on the top of a vertical wall. The point of projection is 8 m above horizontal ground. The ball hits a vertical fence which is at a horizontal distance of 9 m from the wall (see diagram).

(i) Calculate the height above the ground of the point where the ball hits the fence. **[5]**

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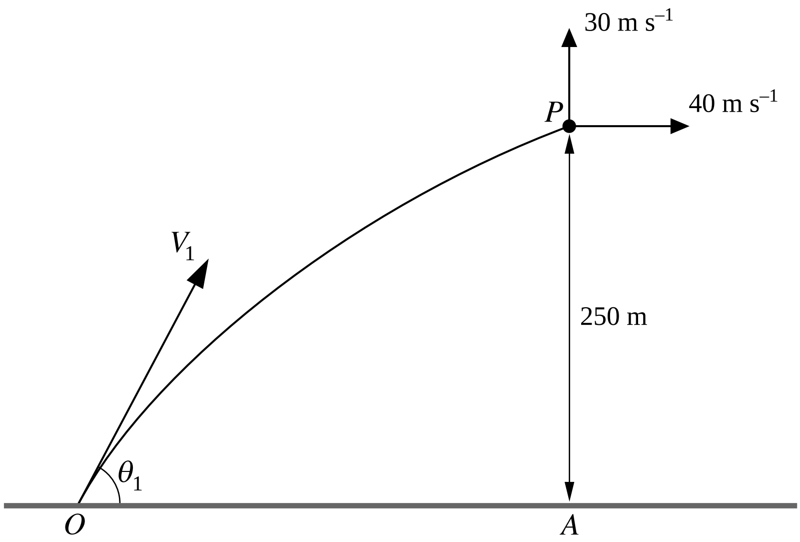
(ii) Calculate the direction of motion of the ball immediately before it hits the fence. **[5]**

(iii) It is given that 30% of the kinetic energy of the ball is lost when it hits the fence. Calculate the speed of the ball immediately after it hits the fence. **[4]**

(Q7, June 2009)

16



A particle P is projected with speed V_1 m s^{-1} at an angle of elevation θ_1 from a point O on horizontal ground. When P is vertically above a point A on the ground its height is 250 m and its velocity components are 40 m s^{-1} horizontally and 30 m s^{-1} vertically upwards (see diagram).

(i) Show that $V_1 = 86.0$ and $\theta_1 = 62.3°$, correct to 3 significant figures. **[5]**

At the instant when P is vertically above A, a second particle Q is projected from O with speed V_2 m s^{-1} at an angle of elevation θ_2. P and Q hit the ground at the same time and at the same place.

(ii) Calculate the total time of flight of P and the total time of flight of Q. **[4]**

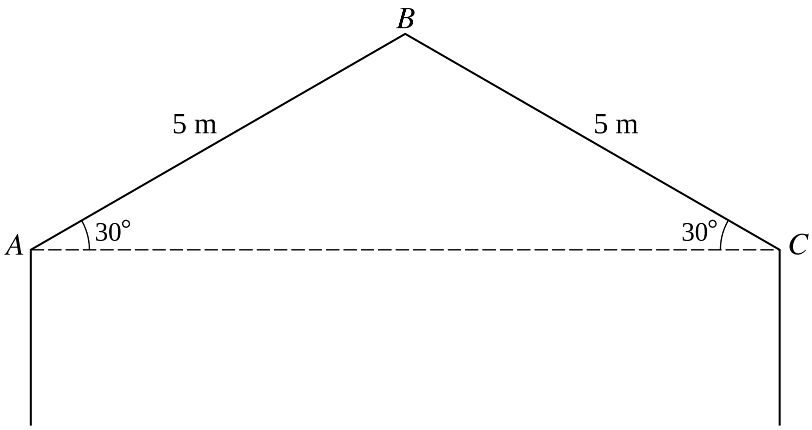
(iii) Calculate the range of the particles and hence calculate V_2 and θ_2. **[8]**

(Q6, Jan 2010)

17 A particle is projected horizontally with a speed of 7 m s^{-1} from a point 10 m above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground. **[6]**

(Q1, June 2010)

18



A small ball of mass 0.2 kg is projected with speed 11 m s^{-1} up a line of greatest slope of a roof from a point A at the bottom of the roof. The ball remains in contact with the roof and moves up the line of greatest slope to the top of the roof at B. The roof is rough and the coefficient of friction is $\frac{1}{2}$. The distance AB is 5 m and AB is inclined at 30° to the horizontal (see diagram).

(i) Show that the speed of the ball when it reaches B is 5.44 m s^{-1}, correct to 2 decimal places. **[6]**

The ball leaves the roof at B and moves freely under gravity. The point C is at the lower edge of the roof. The distance BC is 5 m and BC is inclined at 30° to the horizontal.

(ii) Determine whether or not the ball hits the roof between B and C. **[7]**

(Q7, June 2010)

19 A small ball B is projected with speed 14 m s^{-1} at an angle of elevation 30° from a point O on a horizontal plane, and moves freely under gravity.

(i) Calculate the height of B above the plane when moving horizontally. **[2]**

B has mass 0.4 kg. At the instant when B is moving horizontally it receives an impulse of magnitude I N s in its direction of motion which immediately increases the speed of B to 15 m s^{-1}.

(ii) Calculate I. **[3]**

For the instant when B returns to the plane, calculate

(iii) the speed and direction of motion of B, **[4]**

(iv) the time of flight, and the distance of B from O. **[5]**

(Q6, Jan 2011)

20 A particle is projected with speed 7 m s^{-1} at an angle of elevation of 30° from a point O and moves freely under gravity. The horizontal and vertically upwards displacements of the particle from O at any subsequent time t s are x m and y m respectively.

(i) Express x and y in terms of t and hence find the equation of the trajectory of the particle. **[4]**

(ii) Calculate the values of x when $y = 0.6$. **[4]**

(iii) Find the direction of motion of the particle when $y = 0.6$ and the particle is rising. **[4]**

(Q5, June 2011)

21 A particle P is projected with speed $40\,\text{m}\,\text{s}^{-1}$ at an angle of $35°$ above the horizontal from a point O. For the instant 3 s after projection, calculate the magnitude and direction of the velocity of P. **[5]**

(Q1, Jan 2012)

22 A particle P is projected horizontally with speed $15\,\text{m}\,\text{s}^{-1}$ from the top of a vertical cliff. At the same instant a particle Q is projected from the bottom of the cliff, with speed $25\,\text{m}\,\text{s}^{-1}$ at an angle of $\theta°$ above the horizontal. P and Q move in the same vertical plane. The height of the cliff is 60 m and the ground at the bottom of the cliff is horizontal.

(i) Given that the particles hit the ground simultaneously, find the value of θ and find also the distance between the points of impact with the ground. **[6]**

(ii) Given instead that the particles collide, find the value of θ, and determine whether Q is rising or falling immediately before this collision. **[9]**

(Q7, Jan 2012)

23 A boy throws a small ball at a vertical wall. The ball is thrown horizontally, from a point O, at a speed of $14.4\,\text{m}\,\text{s}^{-1}$ and it hits the wall at a point which is 0.2 m below the level of O.

(i) Find the horizontal distance from O to the wall. **[4]**

The boy now moves so that he is 6 m from the wall. He throws the ball at an angle of $15°$ above the horizontal. The ball again hits the wall at a point which is 0.2 m below the level from which it was thrown.

(ii) Find the speed at which the ball was thrown. **[6]**

(Q4, June 2012)

24 A particle is projected with speed $u\,\text{m}\,\text{s}^{-1}$ at an angle of θ above the horizontal from a point O. At time t s after projection, the horizontal and vertically upwards displacements of the particle from O are x m and y m respectively.

(i) Express x and y in terms of t and θ and hence obtain the equation of trajectory

$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}.$$

[4]

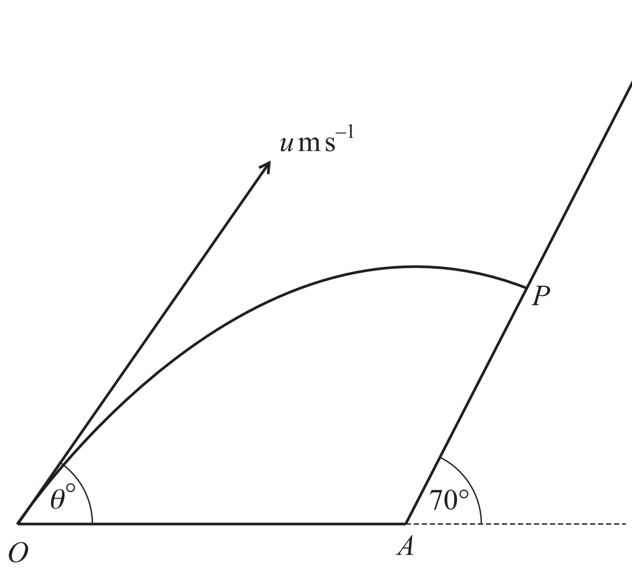
In a shot put competition, a shot is thrown from a height of 2.1 m above horizontal ground. It has initial velocity of $14\,\text{m}\,\text{s}^{-1}$ at an angle of θ above the horizontal. The shot travels a horizontal distance of 22 m before hitting the ground.

(ii) Show that $12.1\tan^2\theta - 22\tan\theta + 10 = 0$, and find the value of θ. **[5]**

(iii) Find the time of flight of the shot. **[2]**

(Q7, Jan 2013)

25



The diagram shows a surface consisting of a horizontal part OA and a plane AB inclined at an angle of 70° to the horizontal. A particle is projected from the point O with speed $u\,\text{m}\,\text{s}^{-1}$ at an angle of $\theta°$ above the horizontal OA. The particle hits the plane AB at the point P, with speed $14\,\text{m}\,\text{s}^{-1}$ and at right angles to the plane, 1.4 s after projection.

(i) Show that the value of u is 15.9, correct to 3 significant figures, and find the value of θ. **[7]**

(ii) Find the height of P above the level of A. **[3]**

The particle rebounds with speed $v\,\text{m}\,\text{s}^{-1}$. The particle next lands at A.

(iii) Find the value of v. **[5]**

(iv) Find the coefficient of restitution between the particle and the plane at P. **[1]**

(Q7, June 2013)